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What supports an aeroplane? Force, momentum, energy and power in flight

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What supports an aeroplane? Force, momentum, energy and power in flight

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Abstract
Some apparently confusing aspects of Newton's laws as applied to an aircraft in normal horizontal flight are neatly resolved by a careful analysis of force, momentum, energy and power. A number of related phenomena are explained at the same time, including the lift and induced drag coefficients, used empirically in the aviation industry.

Introduction

It is widely believed that Newton's laws of motion (Newton 1687) are invalidated in everyday experience because of the forces of gravity and friction. The attention of students is consequently focused on examples of sub-atomic particles and astronomical bodies where friction forces are conveniently negligible. Unfortunately, this creates the incorrect impression that Newton's laws apply imperfectly to everyday objects when friction and gravity are present, ergo Newton's laws never apply in the 'real world', ergo mechanics is a waste of time, whereas in fact they apply universally and their role in the presence of friction and other unintuitive forces has some very interesting consequences.

Normal horizontal flight

As is well known, and conventionally illustrated in diagrams like figure 1, an aircraft is acted upon by forces of thrust (F_T), lift (F_L), drag (F_D) and weight (F_W). In this diagram a dashed surface is shown enclosing the entire aircraft to emphasize that we shall regard it as a single object or 'free body', to which Newton's laws will be applied. If the velocity is constant (i.e. if and only if the aircraft travels in a straight line at constant speed), thrust is equal to drag and lift is equal to weight.

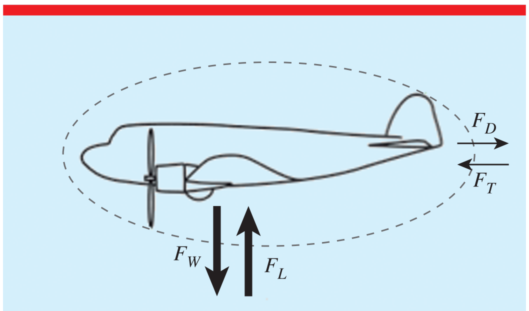

Figure 1. Forces on an aeroplane in normal horizontal flight. 'F' denotes force and the subscripts T, L, D and W denote thrust, lift, drag and weight respectively.

This is a practical example of Newton's first law of motion: *a body in motion will remain in motion in a straight line at a constant velocity if it is acted upon by zero net force*. It does not, however, explain how the forces come to be equal

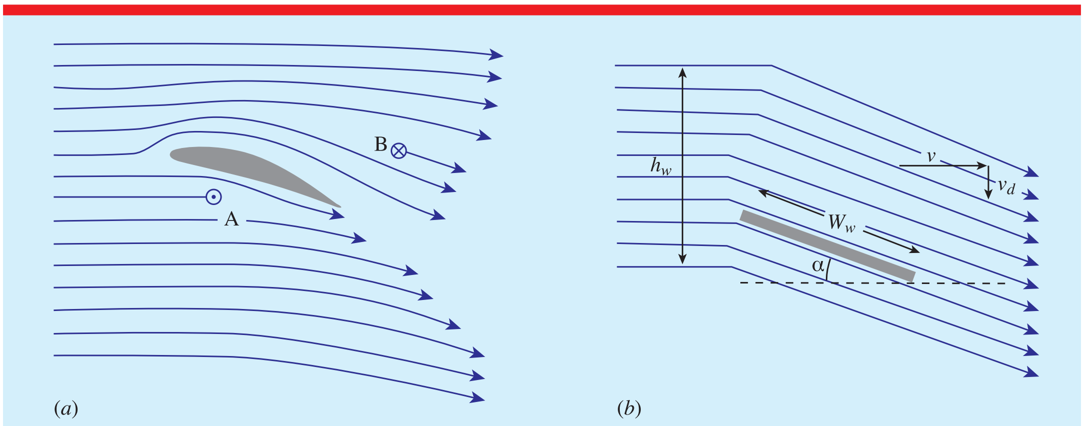

Figure 2. Streamlines over a thin rectangular section wing: (a) conceptualized, (b) idealized.

in the first place, or why the aeroplane would slow down and fall out of the sky if the engines stopped.

In everyday life we each experience downwards force due to gravity but do not accelerate downwards so long as there is a floor or a chair providing an exactly equal upwards force. How does the chair *know* how much upwards force to supply? The answer is that the chair is effectively a spring: in keeping with Hooke's law it is elastically deformed and provides progressively more and more upwards force as the deformation increases, until an equilibrium is established. Likewise the floor, which is a much harder spring, is very slightly deformed by the chair and its occupant, and so on down to the centre of the Earth.

The air, however, is not a spring. Let us now look carefully at Newton's laws in relation to the aeroplane. Far from being invalidated by the presence of gravity and friction, they remain exact, and for heavier-than-air flying bodies this has some very interesting consequences.

Identifying the reaction forces

Newton's third law states that *to every action there is an equal and opposite reaction*. This statement is both simple and profound. It is also exact and universal. In this statement 'action' and 'reaction' are simply forces. They are vectors, so they have direction as well as magnitude. The nomenclature is reversible. Which force is called the 'action' and which the 'reaction' depends on your point of view.

It is sometimes said that in normal horizontal flight the drag force is equal to the thrust force, and the lift force is equal to the weight force because they are action–reaction pairs. This statement is incorrect, on three grounds. Firstly, in terms of Newton's third law, 'action' and 'reaction' forces are *exactly* equal and opposite, *always*, even when acceleration is taking place. However, thrust versus drag and lift versus weight forces clearly have to be unequal in order for the aeroplane to accelerate, rise, turn, follow a constant-altitude flight path over the curved Earth, descend, and slow down again, without which manoeuvres aeroplanes would in practice be useless. Secondly, the action and reaction forces must act on different objects ('free bodies'). Thirdly, the action and reaction forces are always of the same kind. In the case of an aircraft, the weight force is gravitational; it is the force exerted by the Earth upon the airframe. The reaction force must also be gravitational, and this rules out the lift force as a candidate. In fact, the reaction to the weight force is an attractive gravitational force exerted by the airframe upon the Earth.

The lift force and its associated reaction force

The manner in which the wing interacts with the air is the subject of endless discussion. Figure 2(a) illustrates how the streamlines diverge around the leading edge of the aerofoil section and merge at the trailing edge. Two aspects of the aerofoil should be noted: (a) the slab of the wing has a positive 'angle of attack' (α) to the airstream, and/or (b) the overall shape of the aerofoil is asymmetrical. It is found experimentally that the air velocity is greater over the upper surface than the lower, and that the pressure, which is non-uniform, is less above the wing than below it. As discussed by Babinsky (2003), several physical effects come into play, for example the Bernoulli effect (*where total energy is constant, fluid pressure diminishes as its velocity increases*), the Coanda effect (*that fluid flow tends to follow surfaces*) and what we might call the pressure-gradient effect, whereby pressure decreases towards the centre of curvature for curved streamlines, owing to the inertia of the fluid.

Fortunately, the details of these processes are not critical to our discussion. The net effect of all these interactions is that the air exerts an upwards force on the wing (the lift) and the wing exerts a downwards force on the air. (Note that the air at low pressure which sucks the wing upwards is itself sucked downwards by the wing.) These are a true action–reaction pair in the sense of Newton's third law of motion.

The aeroplane does not accelerate upwards because, viewing it as a free body, the lift force is generally balanced by the weight force. However the air, viewed as a different free body, does not have any force acting on it to counteract the downwards force from the wings. The air therefore accelerates downwards away from the wing in accordance with Newton's second law of motion: *a force acting on a body produces an acceleration proportional to the force and inversely proportional to the mass of the body*. Now, acceleration is rate of change of velocity and momentum is mass times velocity, so in this situation, where the air body is continuously being refreshed, it is useful to re-express Newton's second law in an equivalent form as *the downwards force on the air is equal to the rate of change of downwards momentum of the air*. Thus, a downwards moving stream of air is created which is called *downwash*.

Downwash is significant. A graphic illustration of this is shown in figure 3. Note that in this photograph the aircraft itself is descending, because the flaps are lowered. Other dramatic images of the phenomenon of downwash can be found on the Internet. Likewise, if you watch an aeroplane pass at low altitude over a body of water you can see the surface of the water disturbed by the downwash.

Figure 3. Downwash, trailing vortices and wing region condensation. Korean Air cargo Boeing 747-4B5, at Amsterdam Schiphol, 2013. Copyright Bart Mozer.

In symbols, if F_L is the lift force and dM the amount of air that is deflected downwards at vertical velocity v_d in time dt, we have

$$F_L = \frac{d(m_v v_d)}{dt} = v_d \frac{dM}{dt}, \qquad (1)$$

where dM/dt is the rate at which air mass is being deflected. The symbol 'd' means a very tiny amount.

Vortices

In figure 2(a) (and in the experimentally obtained figure 4 of Babinsky 2003) a streamline is shown disappearing at 'A' and reappearing at 'B'. This is a real effect whereby air at relatively high pressure beneath the wing leaks around the wingtip to the area of relatively low pressure above the wing. This gives rise to *wingtip vortices*, clockwise about the port wing tip and anticlockwise about the starboard wing tip. In a vortex, a cylinder of air rotates about a line in space, and an equilibrium is established between centrifugal and pressure-gradient forces. Vortices persist because they carry both energy and angular momentum, which are conserved. Blade tip vortices, sometimes leaving helical condensation trails, are likewise produced by propellers and helicopter rotors. As can be seen in figure 3, the wingtip vortices are substantially enhanced by the effect of differential air motion at the margins of the downwash swath to form *trailing vortices*, which are a major feature of the wake of any aircraft and can roll another aircraft over completely if it follows too closely (see, for example, Bradley *et al* 2007). The energy contained in the trailing vortices is derived from the energy originally used to divert air downwards. This energy becomes available as the downwards linear momentum of the downwash becomes dispersed over an ever greater mass of air moving ever more slowly, and kinetic energy (being proportional to velocity squared) is shed. Angular momentum is conserved because the trailing vortex pair is symmetrical.

This description of the association of lift with vortices (via downwash) is consistent with the densely mathematical description contained in the Lanchester–Prandtl lifting line theory (based on the Kutta–Joukowski theorem) which is outside the scope of this paper but can be found in university and industry aerodynamics texts (e.g. Houghton and Carpenter 2002, Tennekes 2009, Abbott and von Doenhoff 1959). Areas of apparent inconsistency, such as the misleading idea that trailing vortices could by themselves support flying machines or animals, can be traced to overly simplistic interpretation of mathematical constructions contained in the Lanchester–Prandtl theory.

The force on the ground

As the downwash propagates into the atmosphere at large, energy will be dissipated in the form of turbulence (including vortices) and ultimately as heat. However, the downwards component of linear momentum will not disappear, because in collisions between physical bodies (such as air masses) *momentum is conserved even if energy is lost*. Ultimately, the momentum will be transferred to the ground in the form of impulse (impulse = force × time = change in momentum). The total impulse applied to the ground will be equal and opposite to the lift impulse that was exerted on the aeroplane by the air in the first place. If the aircraft is flying at a high altitude, this force may be distributed as a (time delayed) very slight increase in pressure over a very large area. Nevertheless, while it is flying, the aircraft is ultimately supported by a reaction force from the ground. Incidentally, even a projectile in free 'ballistic' flight *in vacuo* is ultimately supported by the ground in the sense that the increased impulse received from the ground during launch and impact divided by the flight time exactly corresponds to the force that would have been required to support the same body against gravity if it had been at rest all the time.

Power involved in the lift force

Force is not the same thing as power, and the absence of a net force on the aeroplane does not mean that no power is being used. The amount of power required to accelerate the downwash air is force times distance per unit time, or force times velocity. Therefore, the amount of power (P_L) that must be dissipated (on a continuous basis) to provide the lift force (F_L) is

$$P_L = F_L v_d = v_d^2 \frac{dM}{dt}. \qquad (2)$$

This simple equation has several interesting consequences. Firstly, *no lift is possible without the dissipation of power*. Secondly, the amount of power required for a given amount of lift will be less if the downwash velocity can be made less, as it can, for example, by varying the wing dimensions. However, this comes at the cost of increasing the drag. For this reason fighter aircraft, for which speed is a paramount design consideration, are not built to fly economically. Thirdly, if the downwash velocity is increased suddenly (e.g. by lowering flaps) then the lift force will increase dramatically, which it is useful to know when flying too close to the ground.

Thrust and drag

Having shown that lift and weight are not action–reaction pairs but are made equal only by the skill of the pilot, let us examine thrust and drag in a similar way. Thrust is the reaction to air being accelerated aft-wards by the airscrew or turbine blades. As the aircraft passes through the air it leaves behind it a body of air that has acquired velocity in the aft-wards direction, which may be called *backwash*. This requires energy in a similar way to the generation of lift (where air was accelerated downwards), and the ratio of

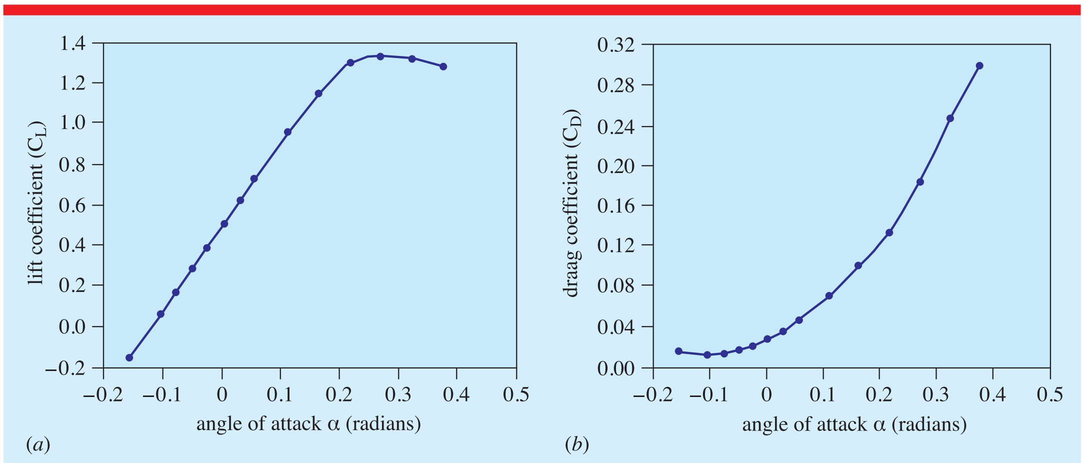

Figure 4. Experimental lift and drag coefficients as a function of angle of attack for a real aerofoil (Gottingen 387), based on Relf and Johansen (1934). 0.1 rad is 5.73°, 0.2 rad is 11.46°.

power required to thrust developed will depend on the size of the propeller disc and the backwash velocity.

At the same time, the air confronting the airframe must be pushed aside continuously to allow the aeroplane to pass. Air has a density of about 1.0 kg m^{-3}, and the mass of air that has to be moved by any vehicle (including a bicycle with its rider) in this way is prodigious. This requires the application of force, and the reaction to this push is called drag. The drag force reaction imparts forwards momentum to the air. After the aircraft has passed there will be a body of air with forwards momentum, which might be called the *slipstream*. (Competitive cyclists routinely ride in the slipstream of others to save energy.) Fairly soon after the aircraft has passed, the backwash and the slipstream will interact: the overall horizontal momentum imparted to the air is zero (for constant velocity flight) and the air will eventually revert to its original state of horizontal motion. However, the energy imparted to the air is not zero. This energy will initially appear as kinetic energy in the form of turbulence and later as a small rise in the temperature of the air.

Lift coefficient

In aviation literature (e.g. Kermode 1970), the above concepts are disguised by being embedded in a family of empirical coefficients and semi-empirical formulae. Of these, the *lift coefficient* (C_L) relates the lift to the wing area (A_w) and airspeed (v),

$$F_L = C_L A_w q, \tag{3}$$

where $q = \rho v^2/2$, the *dynamic pressure*, is the force per unit area required to bring air of density ρ to rest at the rate implied by the velocity. C_L is an empirical function (figure 4) of the angle of attack, a parameter that the pilot is able to measure and control.

We now wish to reconcile our view of flight based on Newton's laws with the empirical view of flight contained in equation (3).

Consider the simple model depicted in figure 2(b). Although it may not be obvious, the *thickness* of the swath of air that is deflected is critical to the amount of momentum the deflected air has and hence the lift, so we need to incorporate this in our model. Figure 3 shows that the wing has an effect on the air over a considerable vertical distance. In reality, of course, the trajectory of the air will be less and less affected with increasing distance from the wing, in a way that depends upon the inertia and viscosity of the air and the dimensions of the wing. However, for the purpose of our simple model, let us suppose that the deflected air forms a uniform swath of thickness h_w. We might guess that h_w depends on the width (technically the 'chord') of the wing, W_w, from leading to trailing edge, so that to a first approximation

$$h_w = KW_w,$$

where K is a dimensionless constant which we expect to have a value somewhere near unity.

With reference to figure 2, we guess that $v_d = v \tan\alpha$, where v is the airspeed. If l_w is the span of the wing, equation (1) then becomes

$$\begin{aligned} F_L &= (v\tan\alpha)(\rho v h_w l_w), \\ F_L &= (v\tan\alpha)(v\rho K W_w l_w), \\ F_L &= (2K\tan\alpha)A_W\frac{\rho v^2}{2}. \end{aligned} \tag{4}$$

This has the form of the lift coefficient equation (equation (3), $F_L = C_L A_w q$), where

$$C_L = 2K\tan\alpha \approx 2K\alpha \text{ (rad)}. \tag{5}$$

In other words, our simple model of the behaviour of the wing yields an equation of the same form as the formula that is found in the aeronautical literature defining the lift coefficient. Moreover, the lift coefficient is predicted to be linearly proportional to the angle of attack, and this agrees with the empirical result in figure 4(a). Indeed, C_L versus α curves for a large number of different aerofoil sections (Relf and Johansen 1934, Abbott and von Doenhoff 1959) show only small departures from the curve shown in figure 4(a); therefore, K is largely independent of the exact shape of the aerofoil. The curve does not pass through the origin because the aerofoil section is asymmetrical. Linearity breaks down (disastrously) at the stalling angle when the air flow becomes turbulent, but this is not part of our model.

The slope of the line in figure 4(a) is 4.0 rad^{-1}. This means that the dimensionless coefficient K is 2.00 and the thickness of the swath of air that is deflected to provide lift is about twice the width (chord) of the wing, which is not unreasonable in respect of figures 2(a) and 3 of this paper (in which the air subjected to low pressure and hence deflected is made visible by condensation) and figure 4 of Babinsky (2003). Anderson and Eberhardt (2001) carried out a similar calculation to the above and concluded that the swath (or 'virtual scoop') thickness for a Cessna 175 wing is 7.3 m at cruising speed.

The air spilt at the wingtips (see the section 'Vortices' above) gives rise to an end effect that causes the slope of the line in figure 4(a), and hence K, to decrease as the aspect ratio (span/chord) of the wing decreases. This is the reason why long thin wings are preferred where optimum lift is required, such as in gliders. The diminution is less than about 12% provided that the aspect ratio (effectively l_W/W_w) is more than about four (Abbott and von Doenhoff 1959, figure 2). Equations that are similar to those developed in this paper are presented by Houghton and Carpenter (2002), Linton (2007a, 2007b, 2007c) and incorporate terms involving the aspect ratio to incorporate this effect.

Drag coefficient

In aviation literature (e.g. Kermode 1970), the drag, F_D, is considered to be the sum of the drag due to pushing the airframe through the air, which is called '*form drag*' or '*parasitic drag*', $F_{D,F}$, and the drag that is associated with lift and is called '*induced drag*', $F_{D,I}$,

$$\begin{aligned} F_D &= F_{D,F} + F_{D,I}, \\ F_{D,F} &= C_{D,F}A_F q \qquad \text{and} \\ F_{D,I} &= C_{D,F}A_w q, \end{aligned} \tag{6}$$

where A_F is the cross sectional area presented to the airstream and the other terms were defined in the section 'Lift coefficient'.

Referring again to our simple model in figure 2(b), and equation (4), the amount of power given to the downwash is the lift force times the vertical velocity of the downwash (which is $v\tan\alpha$). The power required is therefore

$$P_L = (v\tan\alpha)F_L. \tag{7}$$

This power comes from the engine via the airscrew or turbine blades, so (ignoring form drag) it must be equal to the induced drag ($F_{D,I}$) times the airspeed,

$$P_{D,I} = vF_{D,I}. \tag{8}$$

Hence, equating P_L with $P_{D,I}$,

$$F_{D,I} = F_L\tan\alpha = C_L A_w q \tan\alpha. \tag{9}$$

This has exactly the form of the induced drag equation (third line of equation (6)), where the induced drag coefficient is

$$C_{D,I} = C_L\tan\alpha \approx 2K\alpha^2 \text{ (rad)}, \tag{10}$$

whereas the expression for C_L was linear in α (up to the stalling speed), the expression for $C_{D,I}$ is quadratic in α, and the quadratic dependence is recognizable in figure 4(b). Furthermore, the value of $C_{D,I}$ is much smaller than C_L, by a factor of $\tan\alpha$. This basically means that the thrust force will, in general, be far less than the weight of the aircraft.

Referring again to figure 4, for $\alpha = 0.1$ (rad) we have $2K = 4.00$ and therefore from equation (10) we would expect C_D to be 0.04. This is to be compared with an observed value of $C_D = 0.068$ (figure 4(b)). More precision is not to be expected because the model is very simple. The important result is that both the form (i.e. dependence on α) and the general magnitude of the lift and drag coefficients are consistent with the expectations based on Newton's laws. Incidentally, this also justifies (at least to a first approximation) the assumptions we made about the swath height being proportional to the chord length and the angle of downwash being equal to the angle of attack.

In fact excellent agreement between our theory and figure 4 is achieved for $\alpha = 0.0$ rad, $\alpha_{eff} = 0.1$ rad, $C_L = 0.4$ and $C_D = 0.02$ if it is noted that: (a) a factor of $1/2$ would appear on the rhs of equation (7) if the continuous increase in downward velocity of air during the accelaration process were incorporated and (b) for an asymmetrical aerofoil the downwash angle, or effective angle of attack, α_{eff}, is offset from the geometrical angle of attack, α. In figure 4 the offset is 0.1 rad.

In any event, when the aircraft load decreases for any reason (e.g. jettisoning payload), lift will exceed weight and the aircraft will accelerate upwards. Conversely, the pilot may decrease the amount of lift required for equilibrium by decreasing the angle of attack. This also reduces the induced drag and the engines can maintain the same airspeed with less power, which is why the engines sound more relaxed (and/or the aircraft flies higher and faster) as a long haul flight proceeds and the fuel load diminishes.

Conclusions

(1) The upwards lift force on the wings is the reaction to a continuous change of downwards momentum of air.

(2) The deflected air comes from a swath whose thickness is of the order of twice the wing chord length.

(3) The continuous change in downwards momentum of the air requires constant application of power.

(4) The effect on the aeroplane of the requirement for power to generate downwash, and hence lift, can alternatively be expressed in terms of the concept of 'induced drag'.

(5) An aircraft is ultimately supported by a weight force distributed over the ground beneath it.

(6) Trailing vortices grow at the downwash margins, conserving energy, linear momentum and angular momentum.

(7) The power dissipated by an aeroplane engine appears (a) partly in air accelerated rearwards, (b) partly in heating and turbulence generated by the interaction of front surfaces of the airframe and the airstream and (c) partly in giving downwards kinetic energy to the downwash. All of this energy is ultimately converted to kinetic energy in turbulence and finally to heat energy.

(8) In summary: the aeroplane wing is very similar in operation to the rotor of a helicopter, the difference being that in a fixed-wing aircraft the necessary high relative-wind velocity is achieved by high forward speed of the aircraft rather than by rotating the lifting surface.

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