Further thoughts on the Solar Constant Frank Thompson

I read with interest the recent paper [1] regarding an experiment to measure the solar constant using a bolometer. The authors admitted that their measurements gave a value much lower than expected.
A re-plot of the data points was undertaken and the graph, Fig 1, is given below

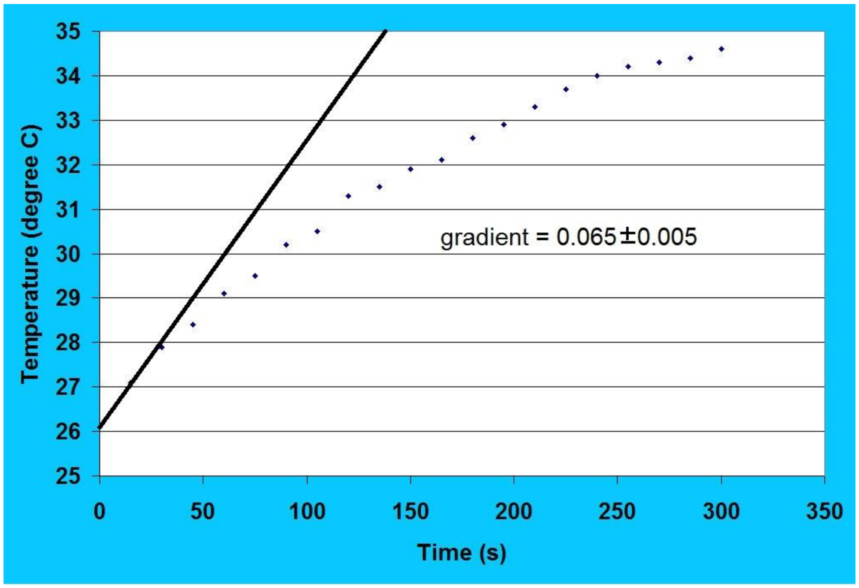

Figure 1 Graph showing the heating of a plate in a bolometer sensor

The graph shows rising temperature values versus time with the plate reaching a final temperature of about 35 degree C. The points fall on a smooth curve which has a logarithmic shape and, in theory, an infinite time is required for the plate to reach its final value.
A theoretical analysis is now given for a solar heated plate.
Assume we have plate of unit area
Let solar constant be I_{SC} Watts
Power loss from cooling is $G(T - T_{amb})$ Watts where T_{amb} is the temperature of the surroundings and T is an elevated temperature.
The parameter G is sometimes called the heat transfer coefficient but it has many different symbols.
The rate of heating of the plate will be equal to the solar power input minus the rate of loss of heat,
If plate has mass, m, and the specific heat of the material is, c, then:

$$mc \frac{dT}{dt} = I_{SC} - G(T - T_{amb}) \quad (1)$$

Substituting a single parameter for ($T - T_{amb}$) allows integration giving:

$$e^{-t/(mc/G)} = \frac{I_{SC} - G(T - T_{amb})}{I_{SC}}$$

Rearranging gives

$$T = T_{amb} + \frac{I_{SC}}{G}(1 - e^{-(G/mc)t}) \quad (2)$$

We see that the final temperature $T_{max} = T_{amb} + I_{SC}/G$

And the initial slope is $I_{SC}/(mc)$ (3)

A value of t which makes the exponential term equal to –1 is also of interest. This time is (mc)/G and it indicates how quickly the temperature is changing. It is called the time constant, τ .

Thus the author's formula [1] only applies to the INITIAL slope of the graph and not the straight-line drawn as a "best-fit" to all the points.

In Fig 1 above, an estimation of the initial slope was found to be 0.065 ± 0.005 K/s which is more than double the slope given in ref. [1].

It must be stressed that at any point after the initial region the solar radiation not only heats up the bolometer but it also heats up the surroundings via the leakage term $G(T - T_{amb})$ in equation (1). For instance, the gradient after 300 seconds is quite small and would give a very small solar constant. This is the Achilles Heel of the method – at the start we have a ***good sensor*** but, as the sensor reaches equilibrium we have a ***bad sensor***.

The value of I_{SC} is then found to be 1080 ±50 W m^{-2} which is a more acceptable value.then the one given by the authors [1] . However, they used all the points for their analysis and therefore their sensor was an average between a good and a bad sensor.
A value of G may also be estimated. Since $T_{max} – T_{amb}$ is 8.5 degree C then the value of G is 125 ±5 W m^{-2} K^{-1} .
As the authors point out, the experiment is interesting in that it lifts physics into an ASTRO-physical regime.
May I add that the experiment requires little time to set up and a thermocouple sensor may be used if an IR thermometer is not available.
Bolometers, of course, have much wider scope then measuring only the solar constant. They are used for detecting any visible or infra red radiation. Indeed bolometers are sometimes used to detect microwave radiation when sensitive bridge arrangements are employed.
In summary, the authors must be complemented for setting up their apparatus out of doors in order to assess the amount of energy coming from the sun.

Additional Information

The Stefan-Boltzmann Formula

On page 4 of the article we see the above formula:

$$Power / m^2 = \sigma T^4 \qquad (4)$$

Figure 4. Heating circuit for bolometer. (a) Realisation the heating circuit.

the bolometer for an ideal black body, we can apply the Stefan–Boltzmann law to both energies per square meter:

$$I_{SC} = \sigma T^4,$$

where $\sigma = 5.67 \cdot 10^{-8}\ \mathrm{W \cdot m^{-2} \cdot K^{-4}}$ is Stefan–Boltzmann constant and T is the thermodynamic temperature of the bolometer in thermal equilibrium with its surroundings. Substituting the measured values in the thermodynamic scale, we get the radiation flux value of 508 $\mathrm{W \cdot m^{-2}}$, which is a nice agreement with the previous method.

The power radiated from a source (Unit Area) will be given by this formula and in deep space the temperature falls to almost Absolute Zero.

In the Laboratory we know that objects do not cool down uncontrollably. This is because the surrounding objects, at T_{amb} , are all radiating power to that object. Even if the object was suspended in an evacuated bell jar so that conduction and convection effects were minimized the object would not cool down.

The correct formula is

$$Power / m^2 = \sigma(T^4 - T_{amb}^4) \qquad (5)$$

So an object with the same temperature as that of the surroundings will loose no power by radiation. But if T has a value T_{max} (ie. the stable temperature at the end of the graph) then it will loose heat and this can be calculated:

Power Loss = σ (8.95 – 8.00)10^9 = 53.8 W m^{-2} .

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(a note in passing – the earth is a giant bolometer and the simple formula given in (4) can be used as T_{amb} is only a few degrees K.

The earth intercepts only a fraction of the power emitted from the sun and then re-radiates in all directions
A trivial calculation shows that, if the earth is a perfect radiator, then the equilibrium temperature of the earth is about 300 degree K

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Often a simplified formula is used. The above Stefan formula has a difference of "Squares $(T^2)^2 - (T_{amb}{}^2)^2$ "
Taking T and T_{amb} to be 300 K we have a multiplying factor of 1.08×10^8 and $P = G_{rad}(T-T_{amb})$ where G_{rad} is about 6 W m^{-2} K^{-1}
Now, when $T = T_{max}$ we have 6 x 8.5 which gives 51 W m^{-2} which is almost the same as the correct calculation.

Compared with the total G from experimental findings (about 125 W m^{-2} K^{-1}) the radiation component G_{rad} is small.

The formula which describes the cooling of the bolometer is obtained from the initial equation (1) with the solar term removed.
The result is

$$T - T_{amb} = (T_{\max} - T_{amb})e^{-(G/mc)t} \quad (6)$$

As we can see, at t = 0 then $T = T_{max}$

After an infinite time then $T = T_{amb}$

The value of G depends on radiation, convection and conduction but the main source of cooling is not radiation or conduction but is likely to be convection as hot air over the plate rises and cold air comes in to take its place.
The value of G may be derived from a cooling of the plate which is sheltered from the sun (Newton's Law of cooling).
Then we have

$$I_{SC} = (T_{max} - T_{amb})\, G$$

In the final experiment of [1] the plate is attached to a transistor and this is an arrangement for electrical circuits where Heat Sinks are fitted to power devices.

During the experiment, the voltage measured across the transistor was $U = 6.1\,\mathrm{V}$ and the current was $I = 78$ mA. This power is for the surface area of $S = 9.15\,\mathrm{cm}^2$. Hence, for the value of the solar constant at Earth's surface, we obtain;

$$I_{SC} = \frac{UI}{S} = \frac{6.1\ \mathrm{V} \cdot 78 \cdot 10^{-3}\ \mathrm{A}}{9.15 \cdot 10^{-4}\ \mathrm{m}^2} = 520\frac{\mathrm{W}}{\mathrm{m}^2},$$

which is quite a nice agreement with the results obtained by the two previous methods.

To a reasonable approximation we have

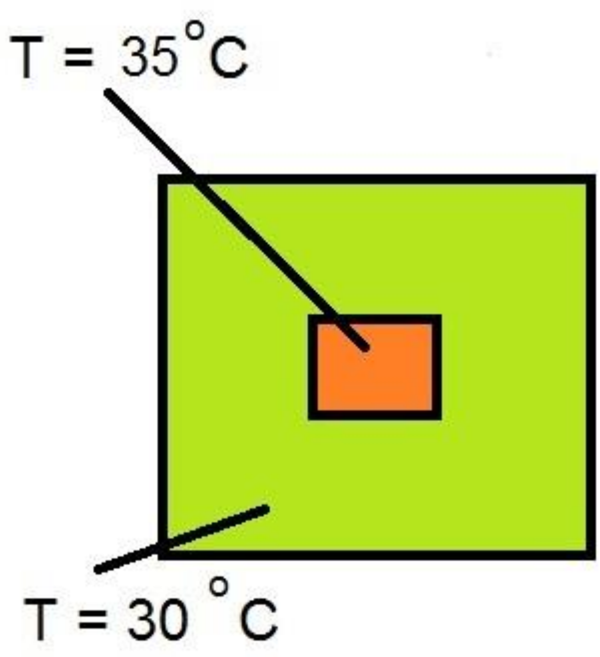
Electrical power in = I xV

And Power loss/ sq m = $G\,(T - T_{amb})$ (7)

These equation replace equation (1) in the original energy balance formula.

It is not known how the plate is mounted but if we imagine only one surface is exposed then we can use the value of G obtained namely, about 125 W m^{-2} K^{-1} .

The plate is likely to have a hot spot where the transistor is mounted as shown:



Perhaps a ¼ of the cell will be at the high temperature and ¾ at the lower temperature
With a temperature rise of about 10 and 5 degree C we may write $9 \times 10^{-4} (1/4 \times 10 + ¾ \times 5)125 = 0.59$ W

This is similar to the electrical power feeding into the transistor.

Without knowing anything about the mounting of the plate at right angles to the solar radiation we would suggest that the loss is about same as the electrical power supplied to the transistor.

The equation (7) may be checked by cooling the plate with the air flow from, say, a hair dryer and recording a fall in the plate temperature with time. As G will be significantly increased the plate should not reach a temperature as high as 34.6 degree C.

Conclusion

It is hoped that this note will clarify points in reference [1] which are misleading.

Reference

[1] L. Brizova and J. Slegr (2017) Estimation of the Solar Constant with a simple bolometer Phys. Ed., 52, p 013008.